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ENGINEERS
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Aerospace Systems Design V

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Flight Control Design using Incremental Nonlinear Dynamic Inversion and Smoothing Estimation

ENGINEERS AUSTRALIA

ROYAL AERONAUTICAL SOCIETY
AUSTRALIAN DIVISION

CONTENTS

INTRODUCTION

- Flight control design for super maneuverable aircrafts require the use of nonlinear approaches such us backstepping, nonlinear dynamic inversion (NDI), adaptive control, robust control and incremental control, to overcome the limitations of linear control techniques. Among them NDI is a suitable controller design approach for fighter aircrafts.

- However NDI has some limitations due to the presence of modeling error and the limited robustness. The incremental form of NDI called incremental nonlinear dynamic inversion (INDI), which a robust version, can be used instead.

- While NDI requires all the model information of the aircraft, INDI uses only the information about the influence of the control (control effectiveness matrix), and any other information related to the aircraft model are obtained through acceleration feedback using on board measurement.

- Even though there exist special sensors that can directly measure angular accelerations from the aircraft, it is still more common the use of IMU sensors that provide angular velocity information. It may not be possible to use expensive sensors to obtained angular acceleration and thus it needs to be estimated.

INTRODUCTION

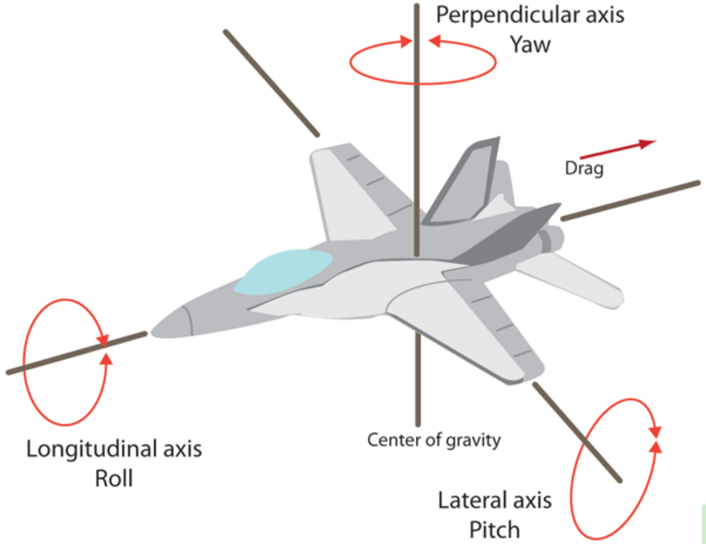
Angular acceleration measurement

- **Direct measurement**

This method refers to the use of special sensors and accelerometers to direct measure angular acceleration signal. This sensors are not yet common and can be expensive.

- **Indirect measurement**

This method refers to the estimation based on analog or digital post processing of available position or velocity signal, using some differentiator or estimator. Some examples are backward differentiators (FIR type), predictive post-filters (IIR type), Kalman filters, composition of neural networks and sliding mode differentiator.



INCREMENTAL NONLINEAR DYNAMIC INVERSION

- Consider the nonlinear system

$$\dot{x} = f(x) + G(x)u$$

- The incremental form of the system can be obtained from the first terms of its Taylor series expansion

$$\dot{x} \approx \dot{x}_0 + \frac{\partial}{\partial x}[f(x) + G(x)u]_{x_0,u_0}(x - x_0) + \frac{\partial}{\partial u}[f(x) + G(x)u]_{x_0,u_0}(u - u_0)$$
$$\approx \dot{x}_0 + \frac{\partial}{\partial x}[f(x) + G(x)u]_{x_0,u_0}(x - x_0) + G(x_0)(u - u_0)$$

were x is the $n \times 1$ state vector, u is the m$\times$ 1 input vector, $f(x)$ is a vector field and $G(x)$ is and $n \times m$ matrix.

- Assuming $x \approx x_0$, since u changes faster than x, we have:

$$\dot{x} = \dot{x}_0 + G(x_0)(u - u_0)$$
$$\dot{x} = \dot{x}_0 + G(x_0)\Delta u$$

- Instead of the control u only, we use the incremental command $\Delta u = (u - u_0)$. Then, applying the inversion the control law is:

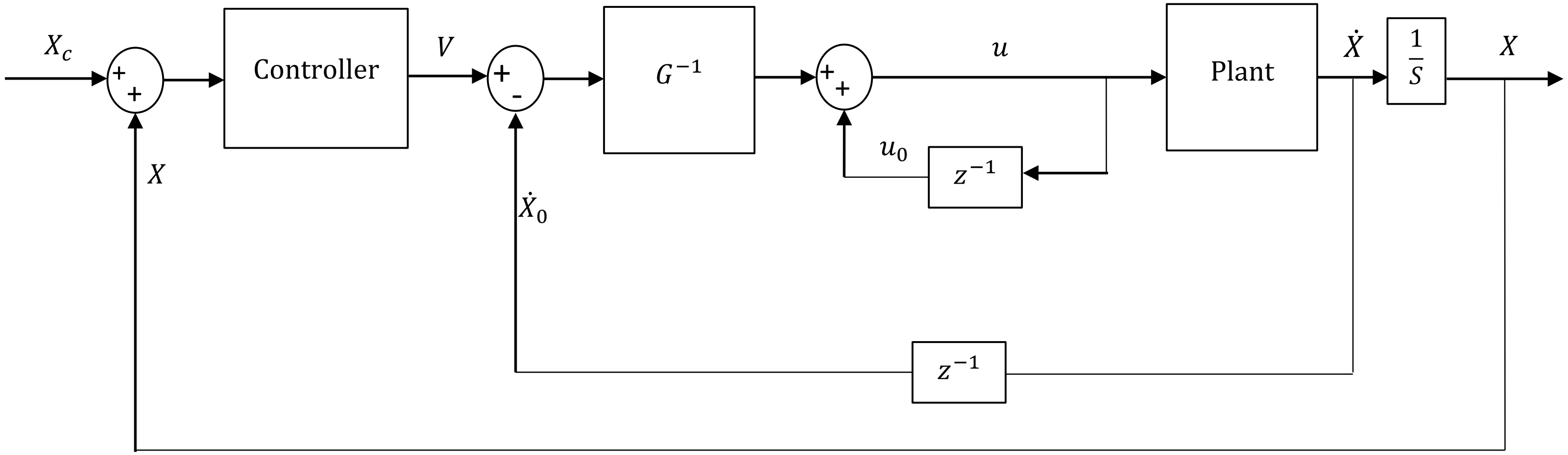
$$u \approx G(x_0)^{-1}(v - \dot{x}_0) + u_0$$

where v is the pseudo input that controls the linearized system thus some linear control approach can be used.

INCREMENTAL NONLINEAR DYNAMIC INVERSION

Pros and Cons

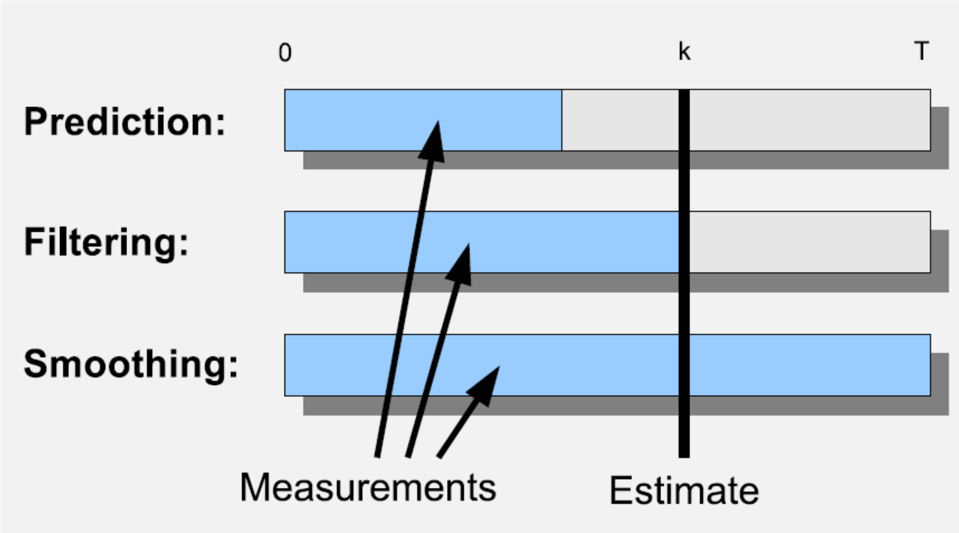
- NDI requires the knowledge of the model of the system $f(x)$ and the effectiveness matrix $G(x)$, while INDI requires only information about $G(x)$ is needed.
- With INDI, even in the presence of uncertainties, the stability can be guaranteed.
- An important issue for INDI implementation is the measurement of the states derivatives that contains biases, noise and delay. It may be also the case that required values cannot be measured and thus they have to be estimated.



Linearized system with INDI

STATE ESTIMATION

- The purpose of an estimator is to provide an estimate of x_K, defined as $\hat{x}_{K/N}$. From the relation of N and K we can have:
- If $N < K$, we are estimating a future value of x_K. This is called **prediction.**
- If $N = K$, we are using all past measurements and the most recent one. This is called **filtering**.
- If $N > K$, we are estimating a previous value of x_K, using past, present and future measurements. This is called **smoothing.**



STATE ESTIMATION: SMOOTHING

- With smoothing we obtain mean-squared estimates $\hat{x}_{k/N}$ of the state vector x_k, where $k < N$.
- Use future measurements to obtain estimates at earlier time points.
- Because more measurements are used to estimate $\hat{x}_{k/N}$ it is expected the estimate to be more accurate and perform better than filters.
- The complexity of the estimation process is increased using smoothing.

We can see three types of smoothing: **Fixed-interval, fixed-point and fixed-lag**

DISCRETE KALMAN FILTER

- Having the discrete signal process model:

$$x_{k+1} = F_k x_k + G_k w_k$$
$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1}$$

Where x_k is the state, F_k is the state transition matrix, z_k is the observation or measurement, H_k is the observation matrix, v_k and w_k are zero mean white gaussian noise with covariance matrices R and Q respectively.

- The Kalman filter equations are given by

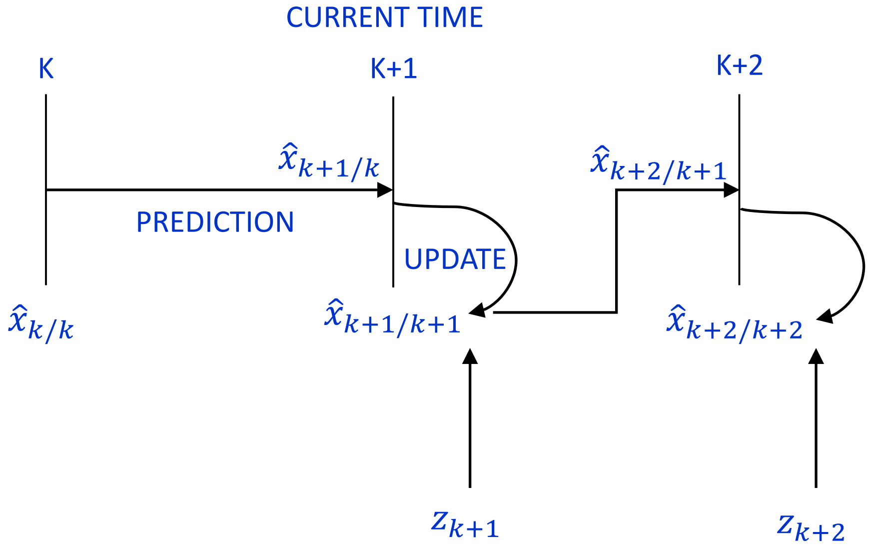
Prediction

$$\hat{x}_{k+1/k} = F_k \hat{x}_{k/k}$$

$$P_{k+1/k} = F_k P_{k/k} {F_k}^T + G_k Q {G_k}^T$$

Update

$$K_{k+1} = P_{k+1/k} H_{k+1}^T (H_{k+1} P_{k+1/k} H_{k+1}^T + R_{k+1})^{-1}$$
$$P_{k+1/k+1} = P_{k+1/k} - K_{k+1} H_{k+1} P_{k+1/k}$$
$$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}(z_{k+1} - H_{k+1} \hat{x}_{k+1/k})$$



- initialize the algorithm with some value for $P_{0/0}$ and $X_{0/0}$

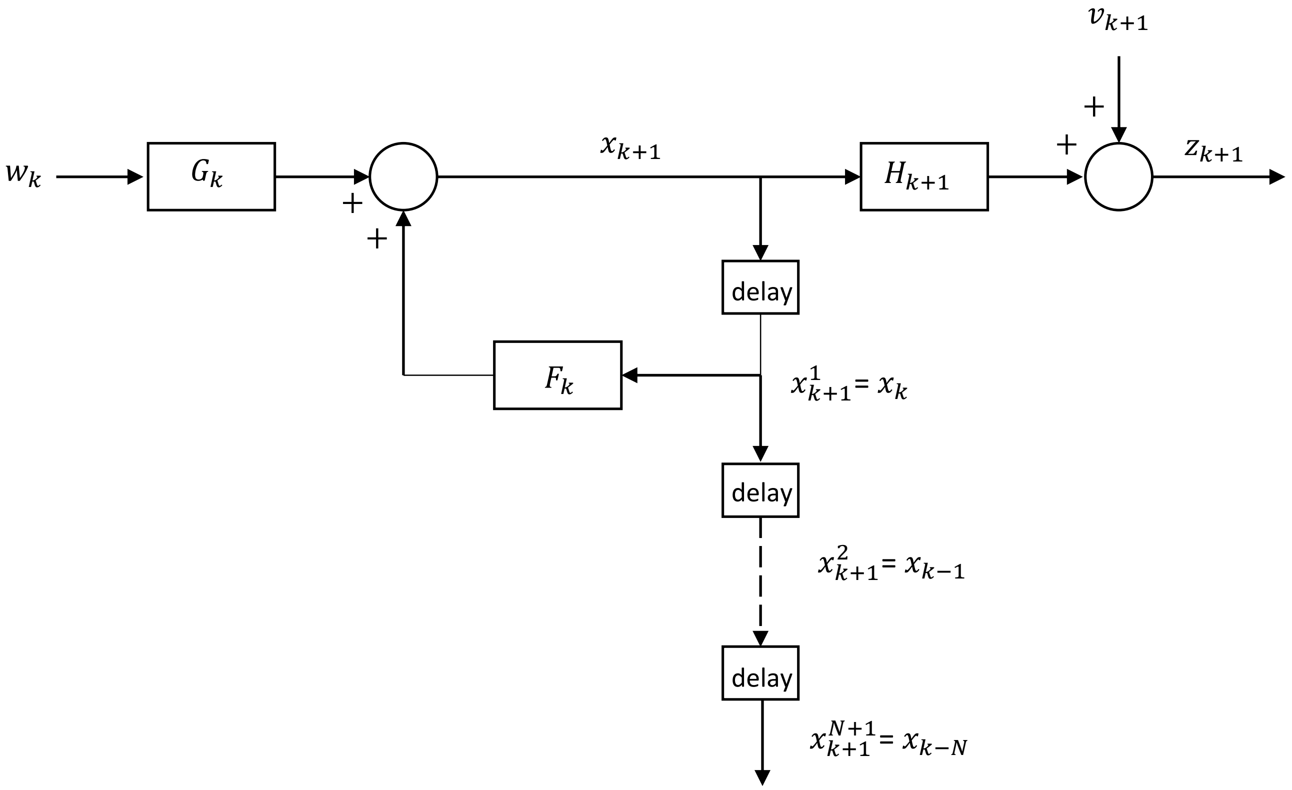
FIXED-LAG SMOOTHING

- By making use of Kalman filter equations, we augment this model and obtain a new model

$$x_{k+1} = F_k x_k + G_k w_k$$
$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1}$$

$$\begin{bmatrix} x_{k+1} \\ x_{k+1}^1 \\ x_{k+1}^2 \\ \vdots \\ x_{k+1}^N \end{bmatrix} = \begin{bmatrix} F_k & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_k^1 \\ x_k^2 \\ \vdots \\ x_k^N \end{bmatrix} + \begin{bmatrix} G_k \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} w_k$$

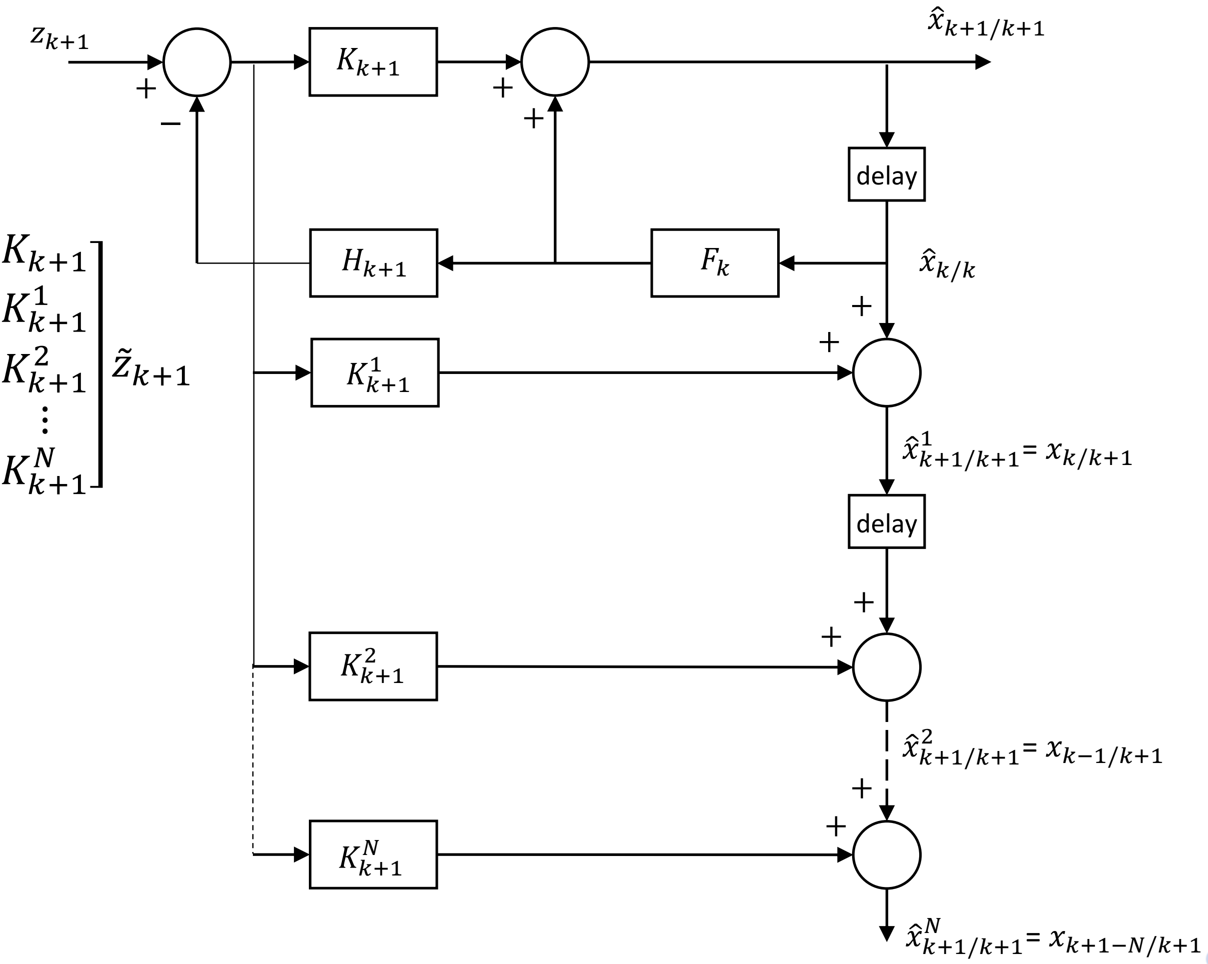
$$z_{k+1} = \begin{bmatrix} H_{k+1} & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ x_k^1 \\ x_k^2 \\ \vdots \\ x_k^N \end{bmatrix} + v_k$$



Augmented signal model

FIXED-LAG SMOOTHING ALGORITHM

- $\tilde{x}_{k+1/k+1} = \tilde{x}_{k+1/k} + \tilde{K}_{k+1}[z_{k+1} - H_{k+1}\hat{x}_{k+1/k}]$
- $\begin{bmatrix} \hat{x}_{k+1/k+1} \\ \hat{x}_{k/k+1} \\ \hat{x}_{k-1/k+1} \\ \vdots \\ \hat{x}_{k+1-N/k+1} \end{bmatrix} = \begin{bmatrix} F_k & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_{k/k} \\ \hat{x}_{k-1/k} \\ \hat{x}_{k-2/k} \\ \vdots \\ \hat{x}_{k-N/k} \end{bmatrix} + \begin{bmatrix} K_{k+1} \\ K_{k+1}^{1} \\ K_{k+1}^{2} \\ \vdots \\ K_{k+1}^{N} \end{bmatrix} \tilde{z}_{k+1}$
- Where $\tilde{z}_{k+1} = [z_{k+1} - H_{k+1}\hat{x}_{k+1/k}]$



FIXED-LAG SMOOTHING ALGORITHM

Augmented Kalman filter equations

$\tilde{x}_{k+1/k} = \tilde{F}_k \tilde{x}_{k/k}$

$\tilde{P}_{k+1/k} = \tilde{F}_k \tilde{P}_{k/k} \tilde{F}_k{}^{T} + \tilde{G}_k Q \tilde{G}_k{}^{T}$

$\tilde{K}_{k+1} = \tilde{P}_{k+1/k} \tilde{H}_{k+1}^{T} (\tilde{H}_{k+1} \tilde{P}_{k+1/k} \tilde{H}_{k+1}^{T} + \tilde{R}_{k+1})^{-1}$

$\tilde{P}_{k+1/k+1} = \tilde{P}_{k+1/k} - \tilde{K}_{k+1} \tilde{H}_{k+1} \tilde{P}_{k+1/k}$

$\tilde{x}_{k+1/k+1} = \tilde{x}_{k+1/k} + \tilde{K}_{k+1}[z_{k+1} - H_{k+1} \hat{x}_{k+1/k}]$

Kalman filter

- **Prediction**

$\hat{x}_{k+1/k} = F_k \hat{x}_{k/k}$

$P_{k+1/k} = F_k P_{k/k} F_k{}^{T} + G_k Q G_k{}^{T}$

- **Update**

$K_{k+1} = P_{k+1/k} H_{k+1}^{T} (H_{k+1} P_{k+1/k} H_{k+1}^{T} + R_{k+1})^{-1}$

$P_{k+1/k+1} = P_{k+1/k} - K_{k+1} H_{k+1} P_{k+1/k}$

$\hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1}(z_{k+1} - H_{k+1} \hat{x}_{k+1/k})$

Smoothing algorithm

- **Prediction**

$\hat{x}^{i}_{k+1/k} = \hat{x}^{i-1}_{k/k}$

$P^{i}_{k+1/k} = P^{i-1}_{k/k} F_k^{T}$

- **Update**

$K^{i}_{k+1} = P^{i}_{k+1/k} H_{k+1}^{T} (H_{k+1} P_{k+1/k} H_{k+1}^{T} + R_{k+1})^{-1}$

$P^{i}_{k+1/k+1} = P^{i}_{k+1/k}(I - H_{k+1}^{T} K_{k+1}^{T})$

$x^{i}_{k+1/k+1} = x^{i}_{k/k} + K^{i}_{k+1}[z_{k+1} - H_{k+1} \hat{x}_{k+1/k}]$

The equations are initialized by $P^{i}_{0/0} = 0$ for all i=1,2,3,…,N

FLIGHT CONTROL DESIGN

Control Augmentation System (CAS) design

- A CAS was designed for the F-18 HARV aircraft with three commands: p_{cmd} and β_{cmd} for lateral-directional mode and α_{cmd} for longitudinal mode.
- the two-time scale separation method is used, where the system is divided into the fast dynamics for angular velocities (p, q, r) and the slow dynamics for the angles (β, α).
- For the inner loop, the moments equations are used:

$$\dot{\omega} = -J^{-1}\omega \times J\omega + J^{-1}\bar{A} + J^{-1}\bar{D}\delta$$

where $\delta = [\,\delta_a \quad \delta_e \quad \delta_r]^T$ is the control input vector, $\dot{\omega} = [\dot{p} \quad \dot{q} \quad \dot{r}]^T$,J is the inertia matrix and $\bar{A}$ and $\bar{D}$ are matrices that contain the aerodynamic coefficients.

- Applying the inversion:

$$\delta = (J^{-1}\bar{D})^{-1}[v_{in} + J^{-1}\omega \times J\omega - J^{-1}\bar{A}]$$

- For the outer loop:

$$\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix} = \begin{bmatrix}\dot{\alpha}_0\\ \dot{\beta}_0\end{bmatrix} + G(x)\begin{bmatrix}q - q_0\\ r - r_0\end{bmatrix}$$

- Applying the inversion:

$$\begin{bmatrix}q\\ r\end{bmatrix} = G(x)^{-1}\left(v_{out} - \begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\end{bmatrix}\right) + \begin{bmatrix}q_0\\ r_0\end{bmatrix}$$

FLIGHT CONTROL DESIGN

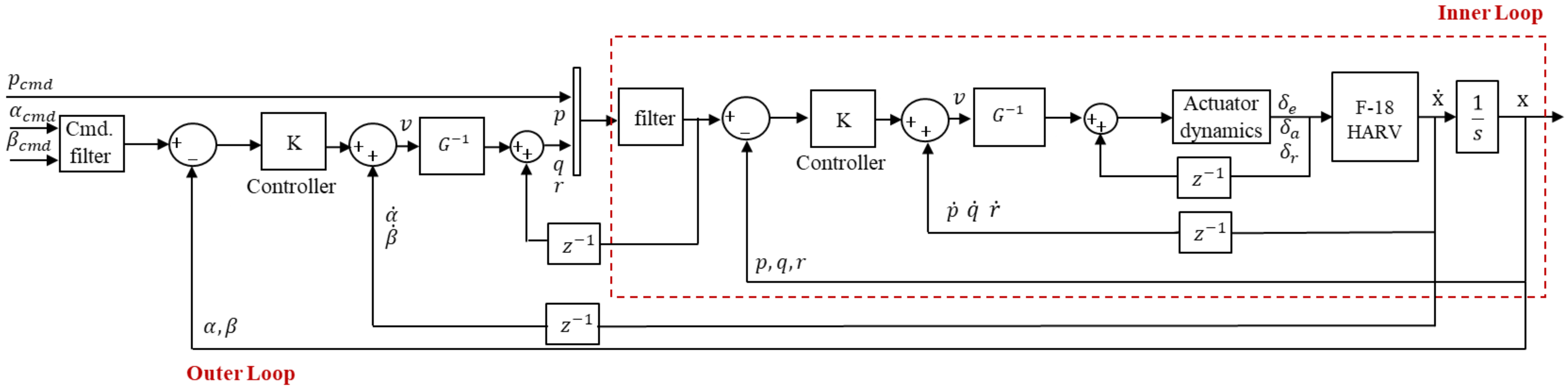
- Parameters and gains:

Surface	Position limit (deg.)	Rate Limit (deg/sec)	Time constant (sec)
Elevator	10.5, - 24	±200	0.0333
Rudder	35, -35	±200	0.0333
Aileron	30, -30	±200	0.0333

Actuators parameters

Parameter		Value
2nd order command filter	damping ratio	0.7
	natural frequency	2.19
Inner loop controller gains	roll rate gain	5
	pitch rate gain	7
	yaw rate gain	5
1st order command filter gains	roll rate gain	5
	pitch rate gain	10
	yaw rate gain	5
outer loop controller gains	angle of attack gain	1.5
	sideslip angle gain	1.5

Gain parameters for INDI



FLIGHT CONTROL DESIGN

Kalman filter estimator

- Our objective is to estimate the angular acceleration.
- To implement the Kalman filter we can define the following system

$$\dot{x} = Ax + w$$
$$y = Cx + v$$
$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ w_2 \end{bmatrix}$$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v$, where $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$

- The discrete form is defined as:

$$x_{k+1} = F_k x_k + G_k w_k$$
$$z_{k+1} = H_{k+1} x_{k+1} + v_{k+1}$$

- Where $F_k = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}, H_{k+1} = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Smoothing algorithm

$$P^i_{k+1/k} = P^{i-1}_{k/k} F^T_k$$
$$K^i_{k+1} = P^i_{k+1/k} H^T_{k+1} (H_{k+1} P_{k+1/k} H^T_{k+1} + R_{k+1})^{-1}$$
$$P^i_{k+1/k+1} = P^i_{k+1/k} (I - H^T_{k+1} \; K^T_{k+1})$$
$$x^i_{k+1/k+1} = x^i_{k/k} + K^i_{k+1} [z_{k+1} - H_{k+1} \hat{x}_{k+1/k}]$$

To compute the true error covariance matrix we take the trace of $\tilde{P}_{k+1/k+1}$

$$P^{i,i}_{k+1/k+1} = P^{i,i}_{k/k} - P^i_{k+1/k} H_{k+1} K^{i^T}_{k+1}$$

The equations are initialized by $P^i_{0/0} = 0$, for all i=1,2,3,…,N

FLIGHT CONTROL DESIGN

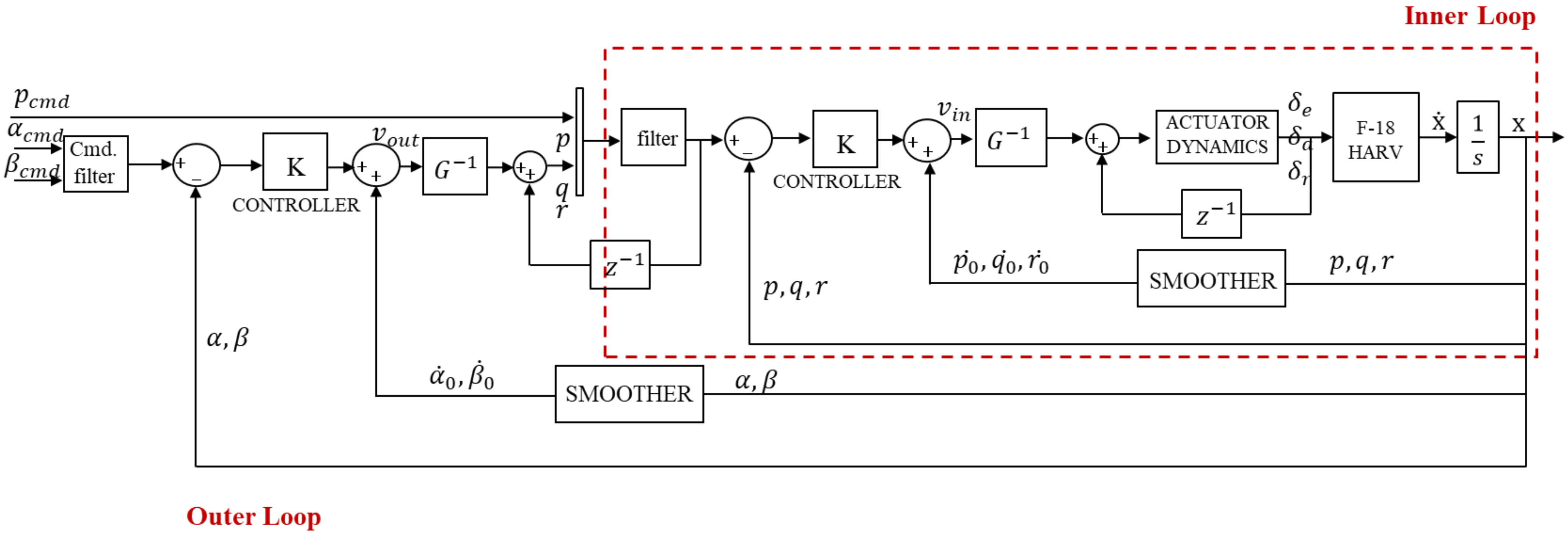


Diagram of INDI with time scale separation and smoothing algorithm

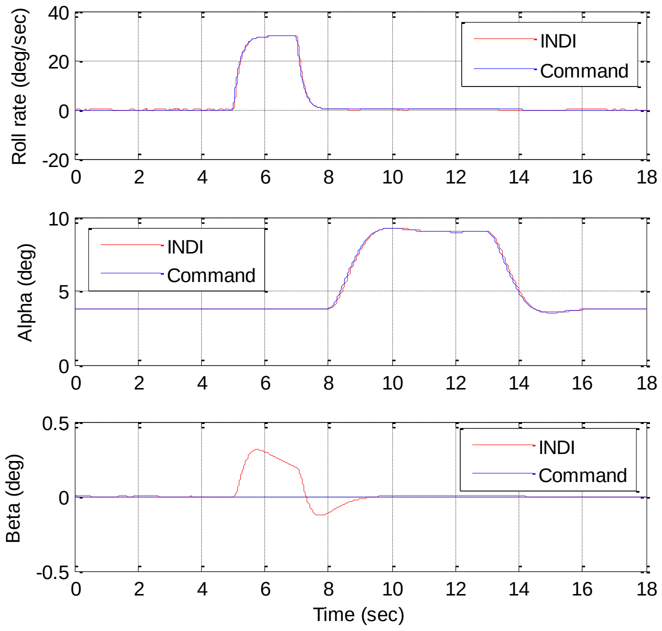
SIMULATION AND RESULTS

- A complex maneuver is implemented for the command, with 30 deg/sec roll rate for two seconds, 9 deg. angle of attack command while the betta command is kept to zero for a proper coordinated turn.

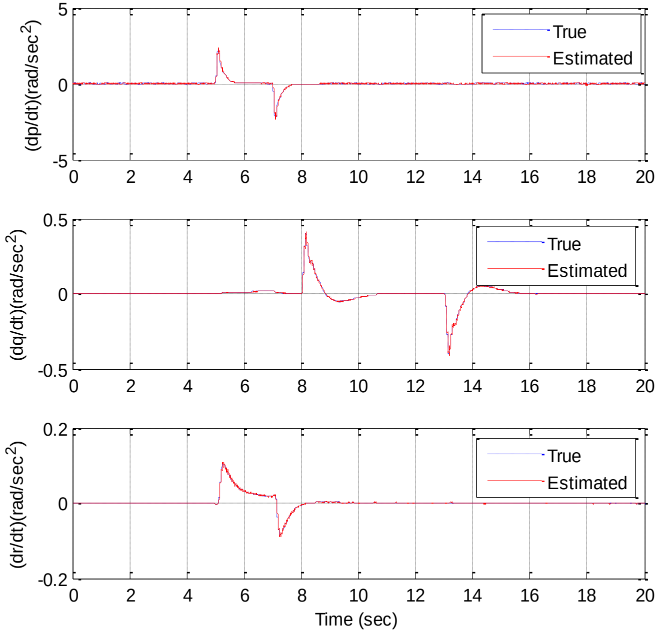
- The sample frequency for the controller is 70 Hz., the IMU sensor works at 1400 Hz. and the sensors for side slip angle and beta angle works at 700 Hz. In this way, the lag for the inner loop smoother is N=20 and for the outer loop smoother the lag is N=10. The power spectral density (PSD) for the IMU sensors is taken as PSD=10^{-9}(deg/sec)^2 1/Hz, and for the α and β sensors this value is PSD=10^{-9}(deg)^{2}1/Hz.

SIMULATION AND RESULTS

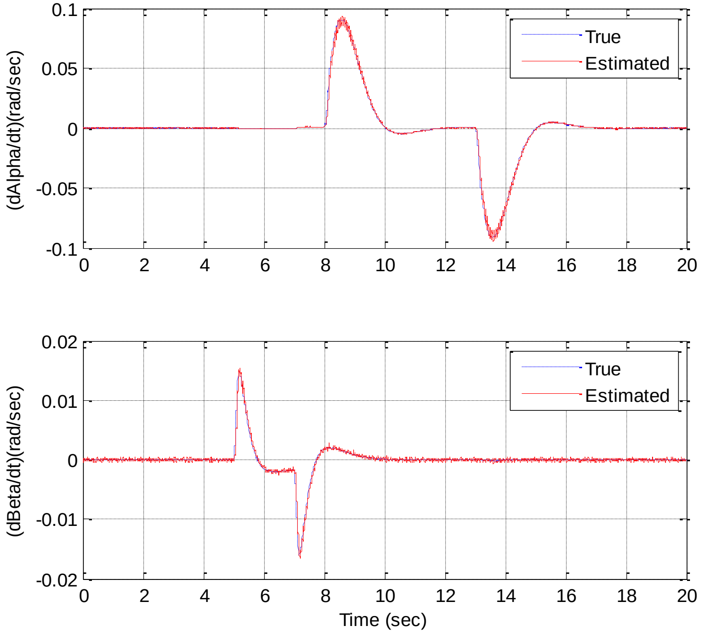
The sample frequency for the controller is 70 Hz., the IMU sensor works at 1400 Hz. and the sensors for side slip angle and beta angle works at 700 Hz. In this way, the lag for the inner loop smoother is **N=20 and for the outer loop smoother the lag is N=10**.



Command Response



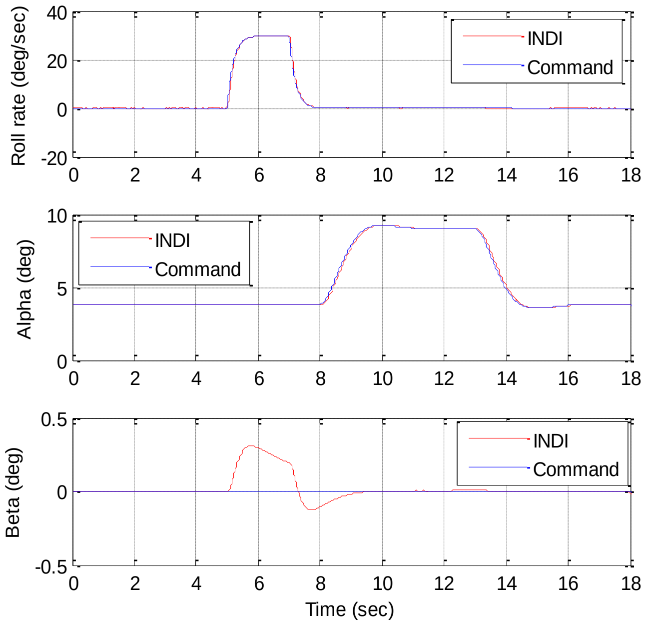
Angular acceleration (inner loop)



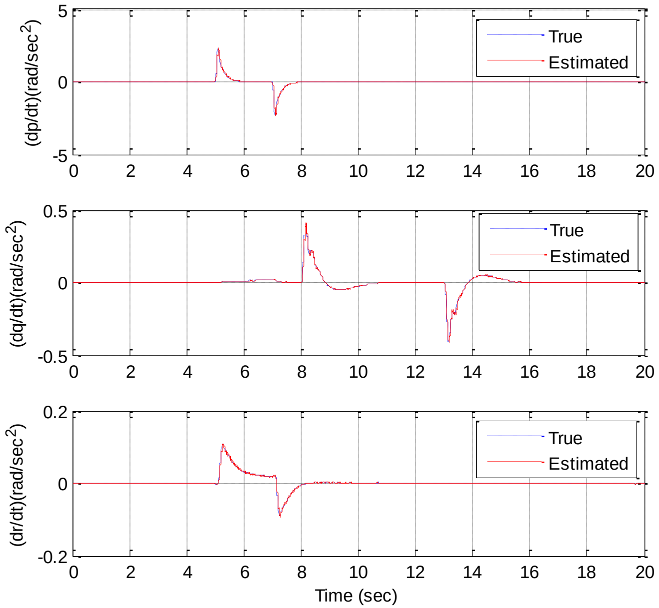
Angular velocity (outer loop)

SIMULATION AND RESULTS

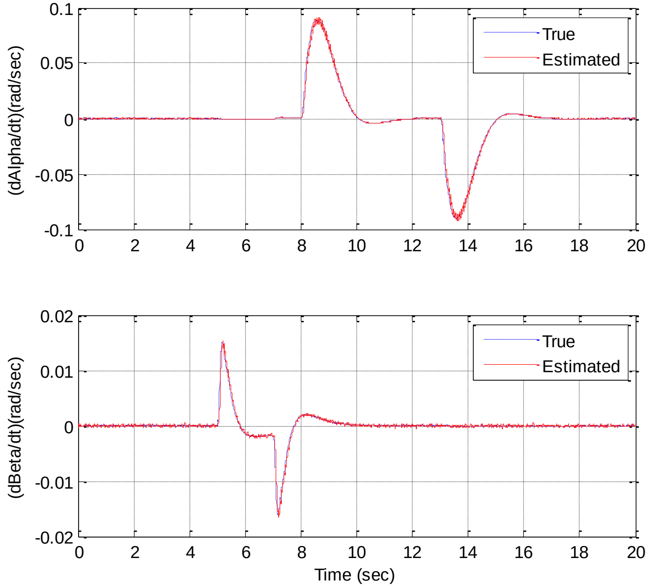
Changing sample frequency to 700Hz for the IMU sensors, and 350 Hz for α and β sensors, the fixed-lag is reduced **to N=10 for the inner loop and N=5 for the outer loop**



Command Response



Angular acceleration (inner loop)

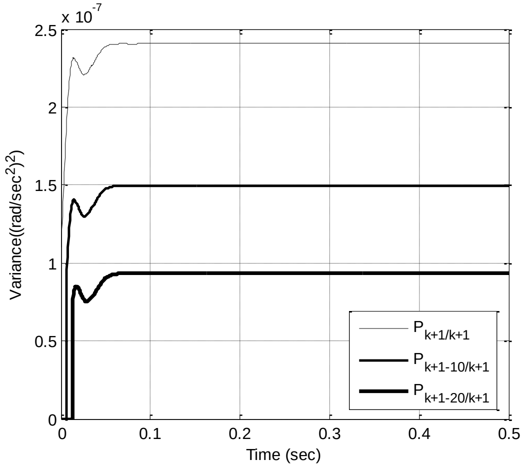


Angular velocity (outer loop)

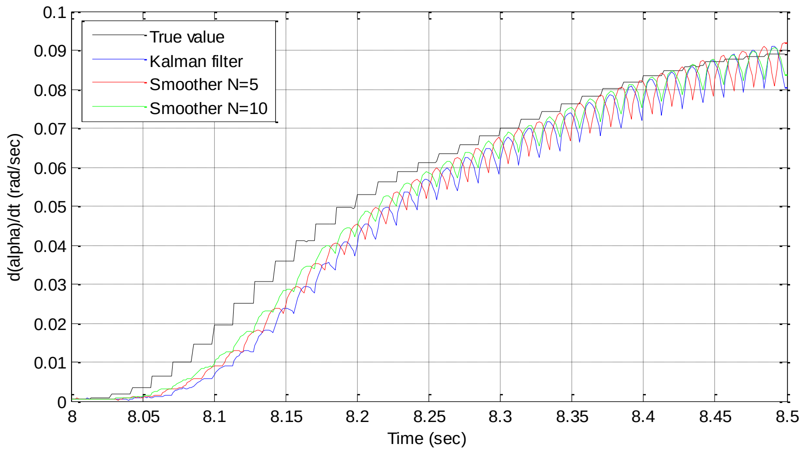
SIMULATION AND RESULTS

Smoother performance:

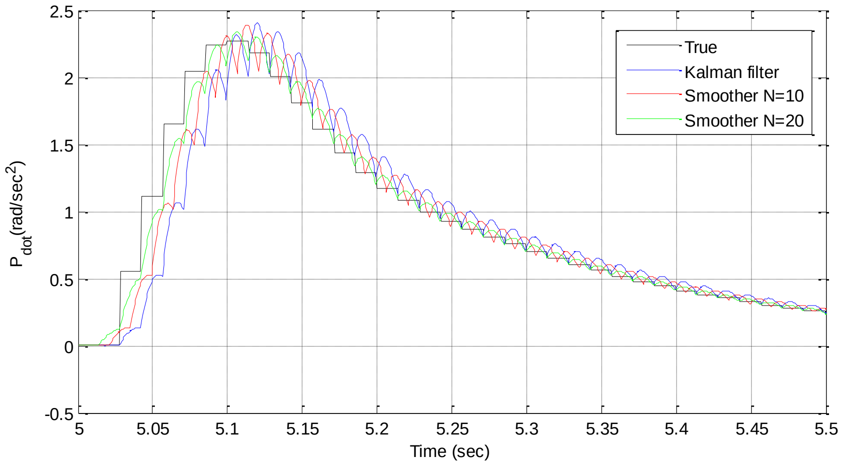
- For the inner loop: when N=5 the improvement is 32.7%, when N=10 the improvement is 55.5 %.
- For the outer loop: when N=10 the improvement is 37.5%, when N=20 the improvement is 61.3 %.



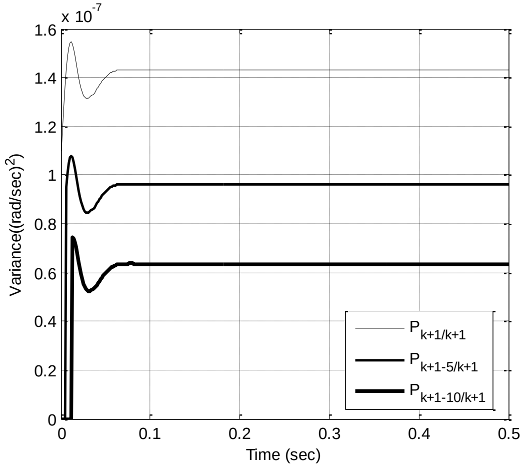
Roll angular acceleration variance



Comparison of angle of attack rate estimation



Comparison of roll angular acceleration estimation



Angle of attack rate variance

CONCLUSIONS

- Incremental nonlinear dynamic inversion approach can provide a robust flight control design for fighter aircrafts.
- Addressing the problem of state derivative availability related to noise and delay problems, a smoother algorithm can handle this problem, using extra available information provided by sensor.
- Additionally, when a direct measurement method using special sensors is employed to obtain angular acceleration or angular velocity, the smoother could still be used as a redundancy system.