Navigation of an Omnidirectional Mobile Rover Using EKF-SLAM Combined with ESO

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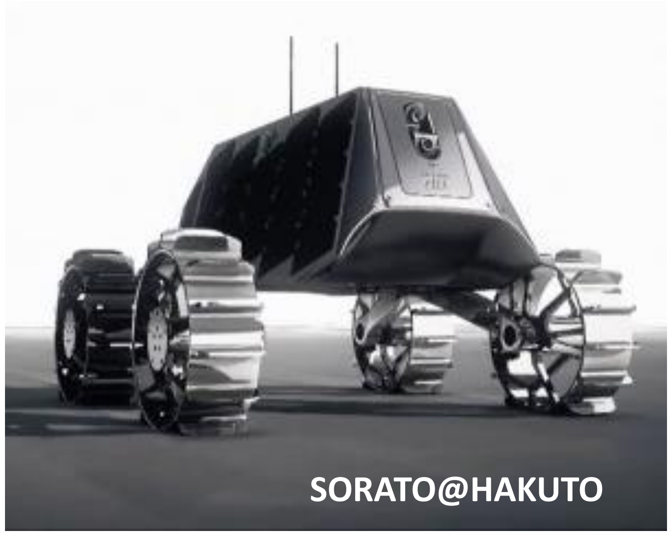
Outline

Background 1: Characteristics of OMR

Conventional Rover



A rover needs to match the direction of movement and its heading angle.

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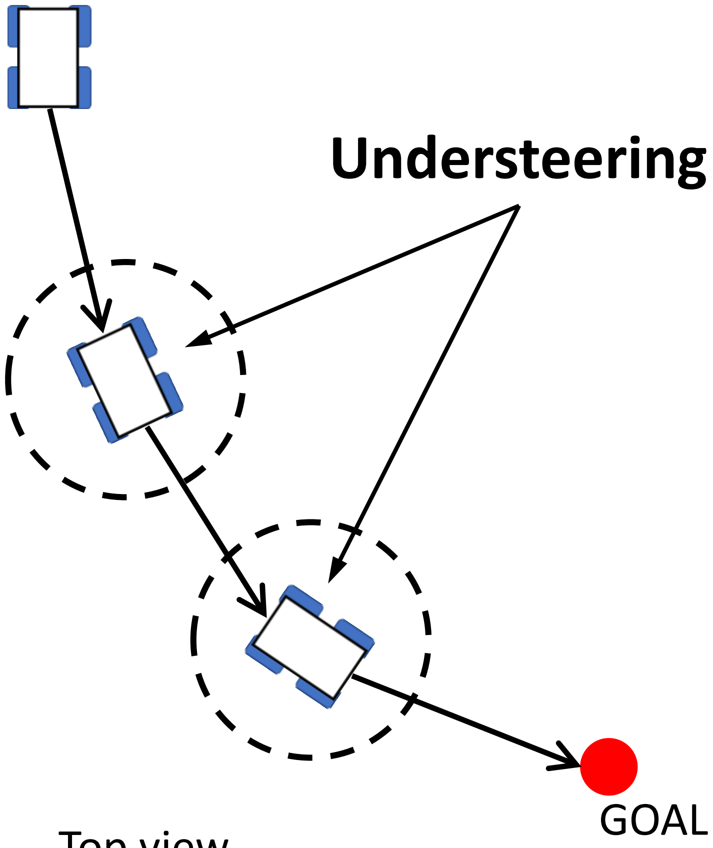
It caused the understeering.

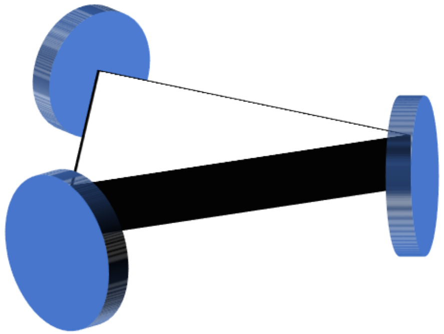


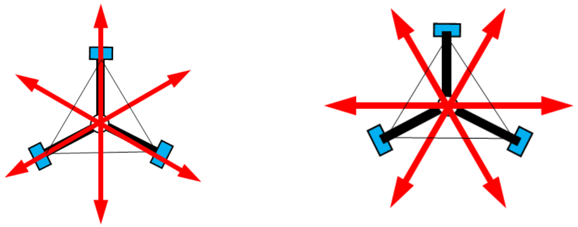
Top view

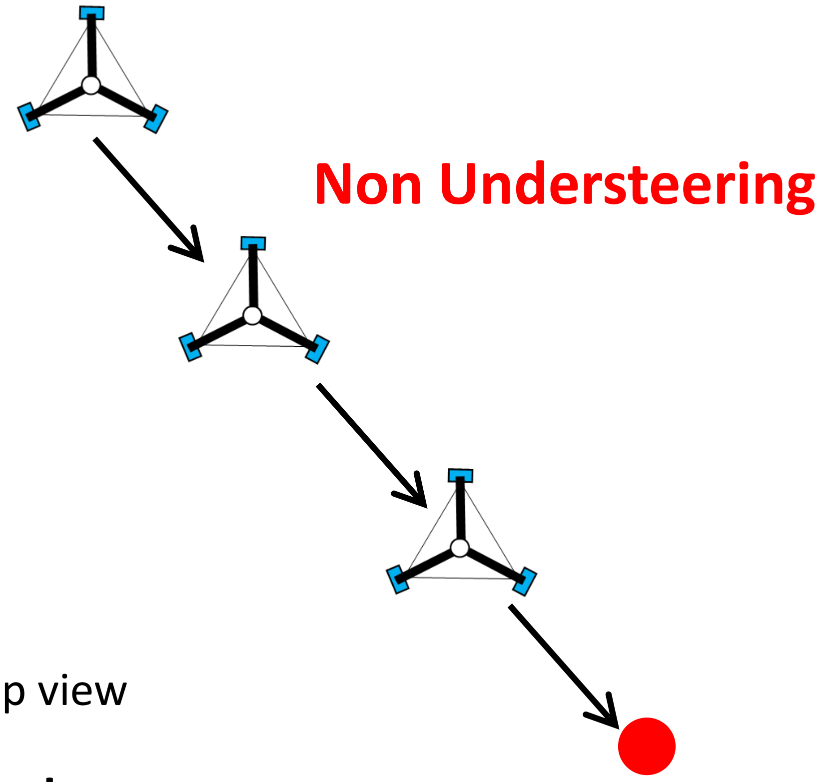
Background 1: Characteristics of OMR

Omnidirectional Mobile Rover


Omni-wheel




Direction of movement

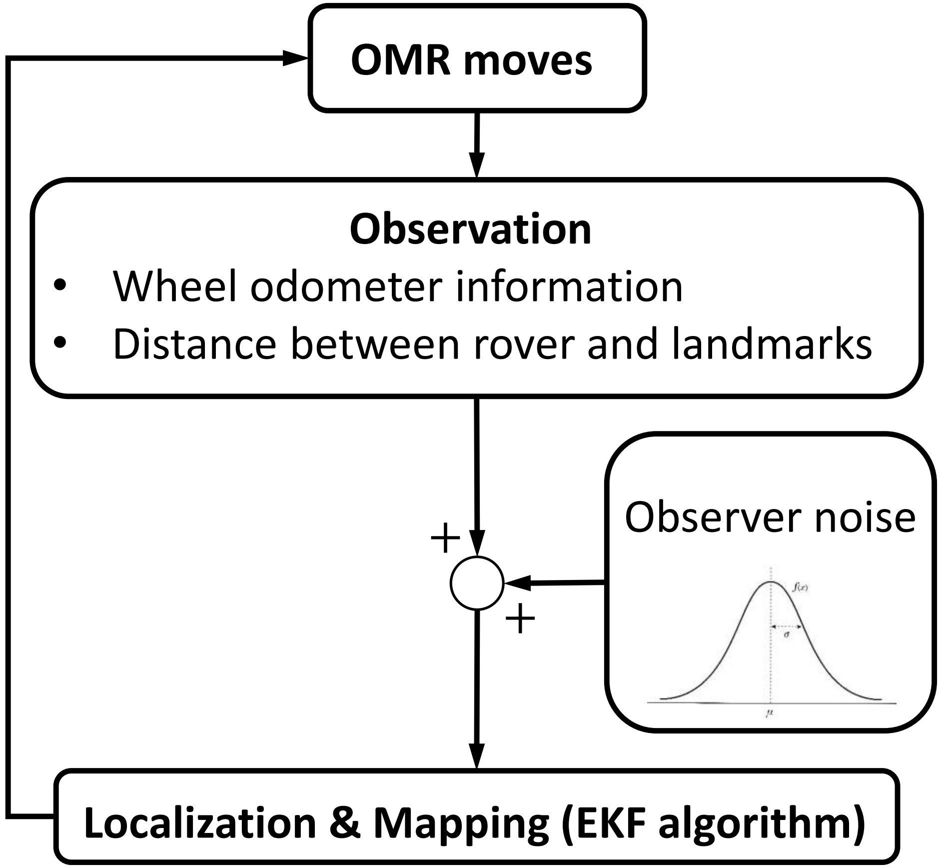


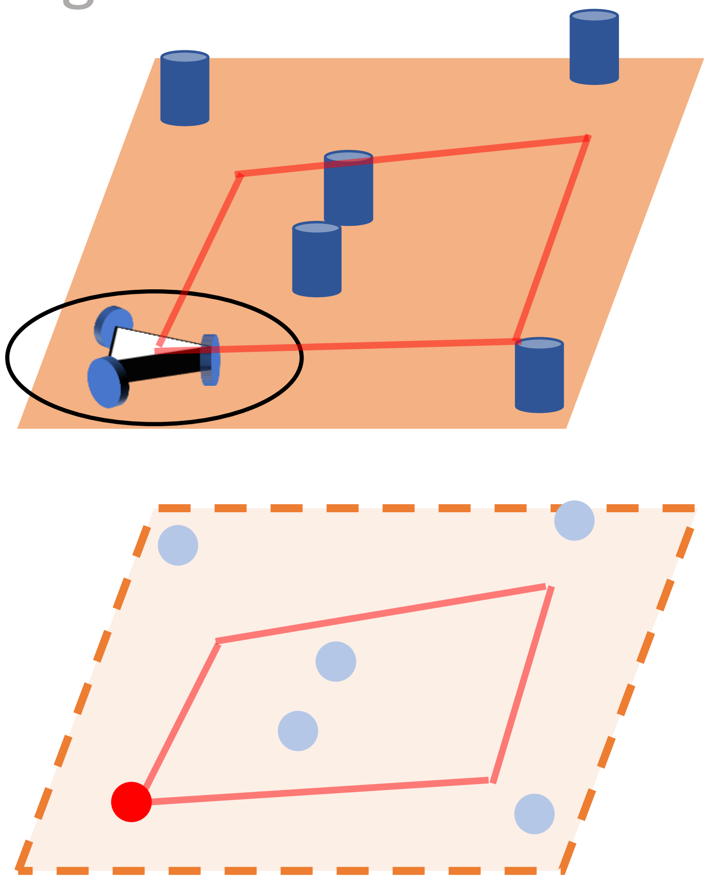
Top view

OMR doesn’t need to change its heading angle.

Background 2: EKF-SLAM problem against disturbances

Extended Kalman Filter for
Simultaneous Localization And Mapping → **EKF-SLAM**





Background 2: EKF-SLAM problem against disturbances

Extended Kalman Filter for

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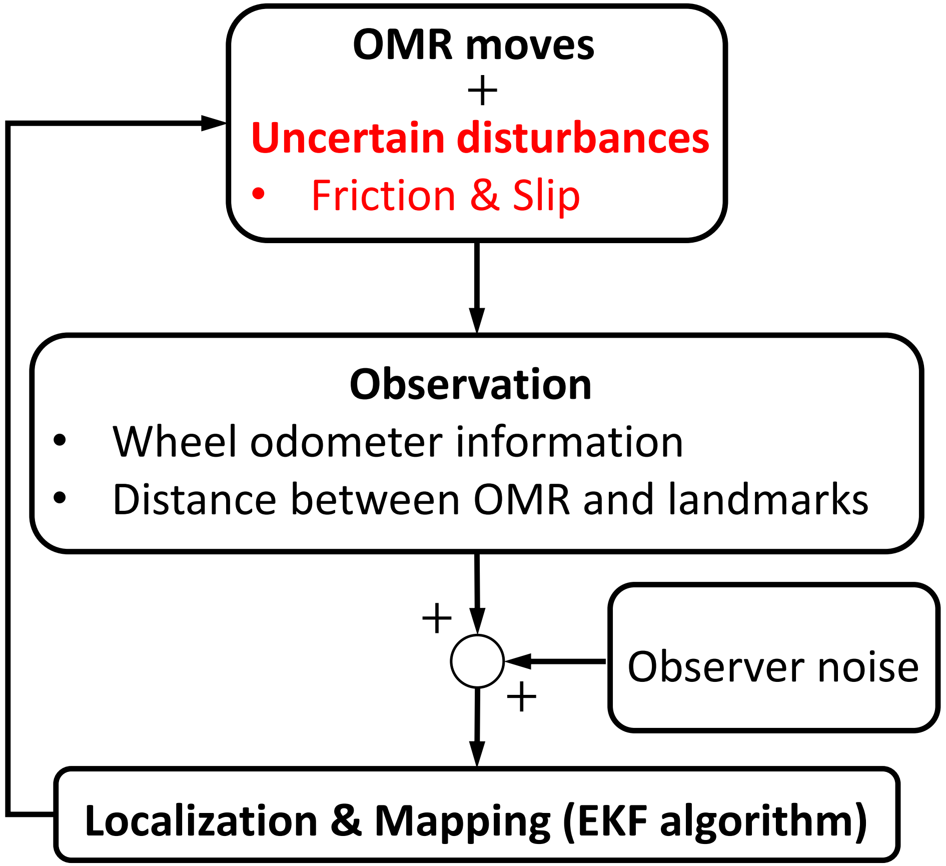
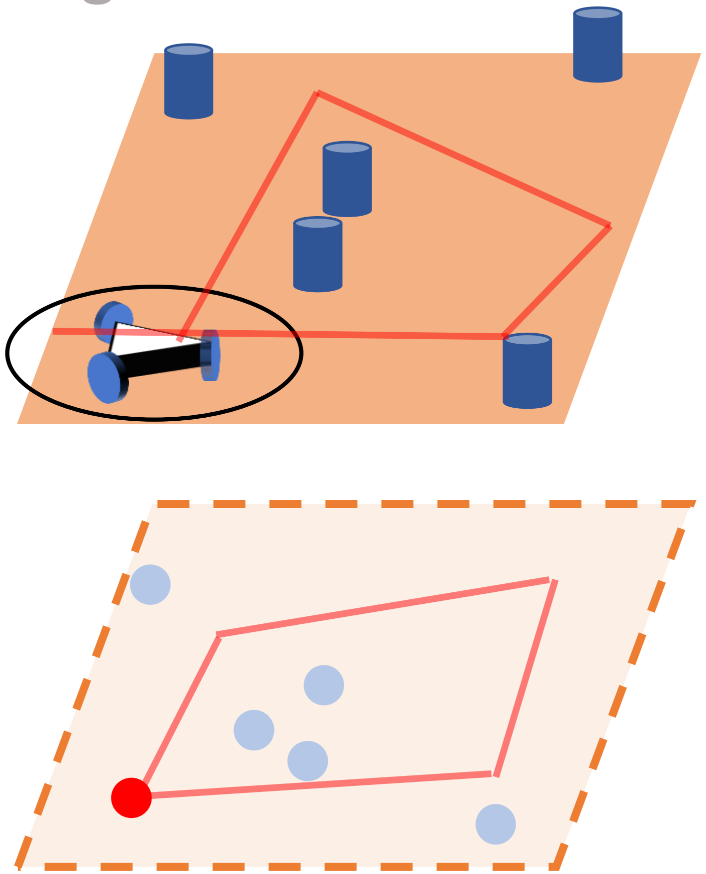




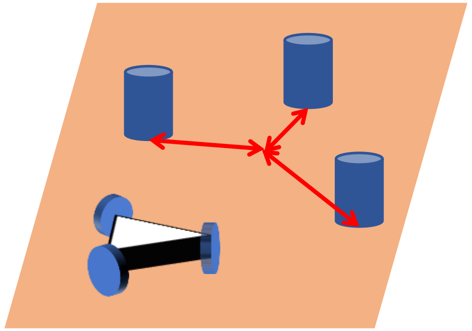
Research objectives

- Proposing EKF-SLAM combined with ESO for OMR with three Omni-wheels.

- Verifying the effectiveness of the proposed method by compare numerical simulation.

Outline

Method 1: EKF-SLAM



State vector

$$\mathbf{X} = \begin{bmatrix} \mathbf{x} \\ \mathbf{Z} \end{bmatrix}$$

Covariance matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{rL} \\ \mathbf{P}_{rL}^T & \mathbf{P}_{LL} \end{bmatrix}$$

$$\mathbf{x}(k) = \mathrm{f}(\mathbf{x}(k-1), \mathbf{u}(k)) + \mathbf{q}(k)$$

$$\mathbf{Z}(k) = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_j \end{bmatrix} = \begin{bmatrix} \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,1}\right) + \mathbf{r}(k) \\ \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,2}\right) + \mathbf{r}(k) \\ \vdots \\ \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,j}\right) + \mathbf{r}(k) \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{\left(x_{L,j} - x\right)^2 + \left(y_{L,j} - y\right)^2} \\ \arctan\left(\dfrac{y_{L,j} - y}{x_{L,j} - x}\right) - \theta \end{bmatrix}$$

Formulation scheme of EKF-SLAM

$$\widehat{\mathbf{X}}(k|k-1) = \mathrm{f}\left(\widehat{\mathbf{X}}(k-1|k-1), \mathbf{u}(k)\right)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}(k-1)\mathbf{P}(k-1|k-1)\mathbf{F}^T(k-1) + \mathbf{Q}$$

$$\mathbf{y}(k) = \mathbf{Z}(k) - \mathrm{h}\left(\widehat{\mathbf{X}}(k|k-1)\right)$$

$$\mathbf{S}(k) = \mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}^T(k) + \mathbf{R}$$

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T(k)\mathbf{S}^{-1}(k)$$

$$\widehat{\mathbf{X}}(k|k) = \widehat{\mathbf{X}}(k|k-1) + \mathbf{K}(k)\mathbf{y}(k)$$

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)$$

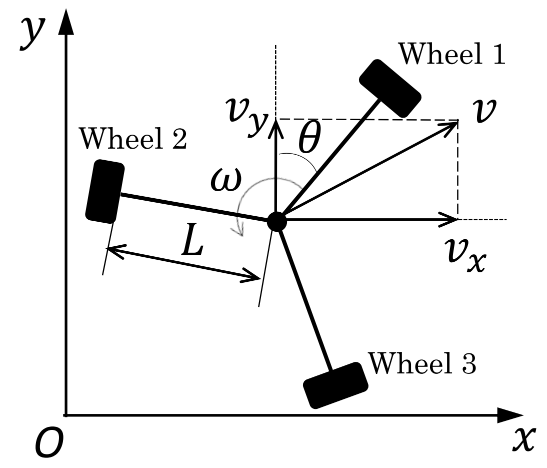
Jacobians

$$\mathbf{F}(k) = \left.\frac{\partial \mathrm{f}}{\partial \mathbf{X}}\right|_{\widehat{\mathbf{X}}(k|k), \mathbf{u}(k+1)}$$

$$\mathbf{H}(k) = \left.\frac{\partial \mathrm{h}}{\partial \mathbf{X}}\right|_{\widehat{\mathbf{X}}(k|k-1)}$$

Method 2: Dynamic model, ESO

The dynamic model of OMR



$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix}, \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} d_x \\ d_y \\ d_\theta \end{bmatrix}$$

$$\mathbf{B} = \frac{1}{2}\begin{bmatrix} -2\cos\theta & \cos\theta + \sqrt{3}\sin\theta & \cos\theta - \sqrt{3}\sin\theta \\ -2\sin\theta & \sin\theta - \sqrt{3}\cos\theta & \sin\theta + \sqrt{3}\cos\theta \\ 2L & 2L & 2L \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_m \end{bmatrix} \qquad \boxed{\mathbf{M}\ddot{\mathbf{x}} = \mathbf{B}\mathbf{u} + \mathbf{d}}$$

m: mass, I_m: inertia moment, L: length

$\because |\mathbf{B}| = 3\sqrt{3}L/2m^2 I_m$
Always $|\mathbf{B}| \neq 0$ is true.
Therefore, matrix $\mathbf{B}$ is regular matrix.

Design method of ESO

$$\begin{cases} \dot{\mathbf{h}}_x = \dot{\mathbf{x}} = \mathbf{h}_v \\ \dot{\mathbf{h}}_v = \ddot{\mathbf{x}} = \mathbf{M}^{-1}(\mathbf{B}\mathbf{u} + \mathbf{d}) = \mathbf{M}^{-1}(\mathbf{B}\mathbf{u} + \mathbf{h}_d) \\ \dot{\mathbf{h}}_d = \dot{\mathbf{d}} \end{cases}$$

$$\mathbf{h}_x \equiv \mathbf{x}, \mathbf{h}_v \equiv \mathbf{v}, \mathbf{h}_d \equiv \mathbf{d}$$

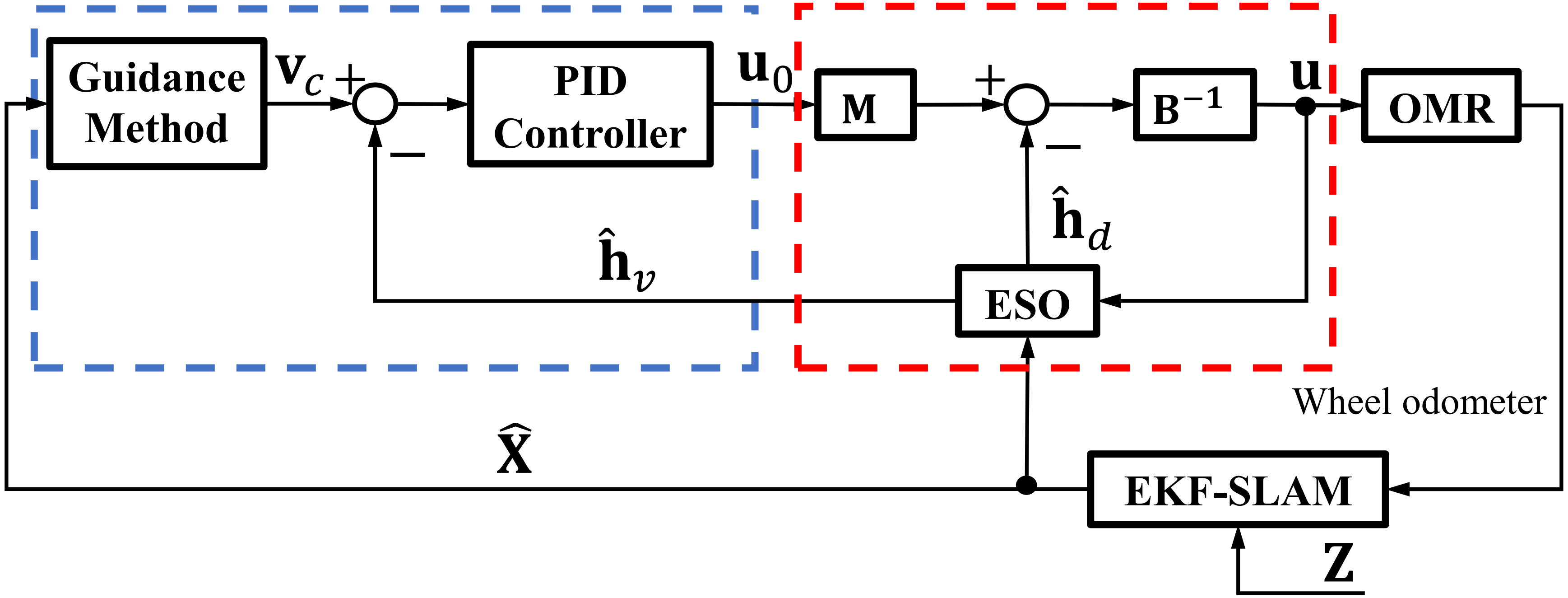
↓

$$\begin{cases} \tilde{\mathbf{x}} = \hat{\mathbf{x}} - \hat{\mathbf{h}}_x \\ \dot{\hat{\mathbf{h}}}_x = \hat{\mathbf{h}}_v + \mathbf{l}_x\tilde{\mathbf{x}} \\ \dot{\hat{\mathbf{h}}}_v = \mathbf{M}^{-1}(\mathbf{B}\mathbf{u} + \hat{\mathbf{h}}_d) + \mathbf{l}_v\tilde{\mathbf{x}} \\ \dot{\hat{\mathbf{h}}}_d = \mathbf{l}_d\tilde{\mathbf{x}} \end{cases}$$

($\mathbf{l}_x, \mathbf{l}_v, \mathbf{l}_d$: observer gain)

Method 3: EKF-SLAM combined with ESO

Block diagram of the proposed method



$$\mathbf{u}_0 = \mathbf{K}_P(\mathbf{v}_c - \hat{\mathbf{h}}_v) + \mathbf{K}_I \sum_{t=0}^{k} (\mathbf{v}_c - \hat{\mathbf{h}}_v) - \mathbf{K}_D \dot{\hat{\mathbf{h}}}_v$$

$\mathbf{K}_P, \mathbf{K}_I, \mathbf{K}_D$: PID gain

$$\mathbf{u} = \mathbf{B}^{-1}(\mathbf{u}_0 - \hat{\mathbf{h}}_d)$$

Compensation

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{B}\mathbf{u} + \mathbf{d}$$

Outline

1. Background

Characteristics of Omnidirectional Mobile Rover (OMR)

EKF-SLAM problem against disturbances

2. Methods

EKF-SLAM

Dynamic model, ESO

EKF-SLAM combined with ESO

3. Results

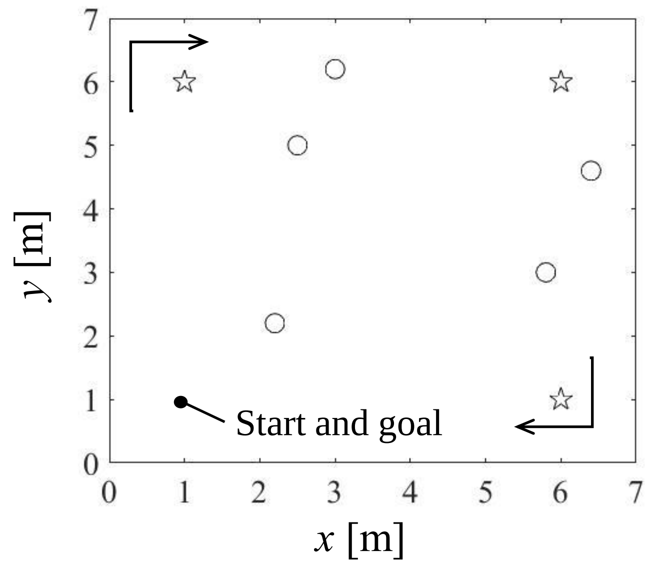
Numerical simulation comparison between EKF-SLAM and EKF-SLAM combined with ESO

4. Conclusion & Future plan

Simulation condition

The OMR physical parameter

Mass: m[kg]	1.5
Moment of inertia: I_m[kg · m^2]	0.025
Length: L[m]	0.10



Ground (○: Landmark, ☆: Waypoint)

Resultant force of each wheel

$$F_i = u_i - \alpha u_i - R_f \ (i = 1,2,3)$$

$$R_f \leq 0.35 \times \mathrm{sgn}(v_i)$$

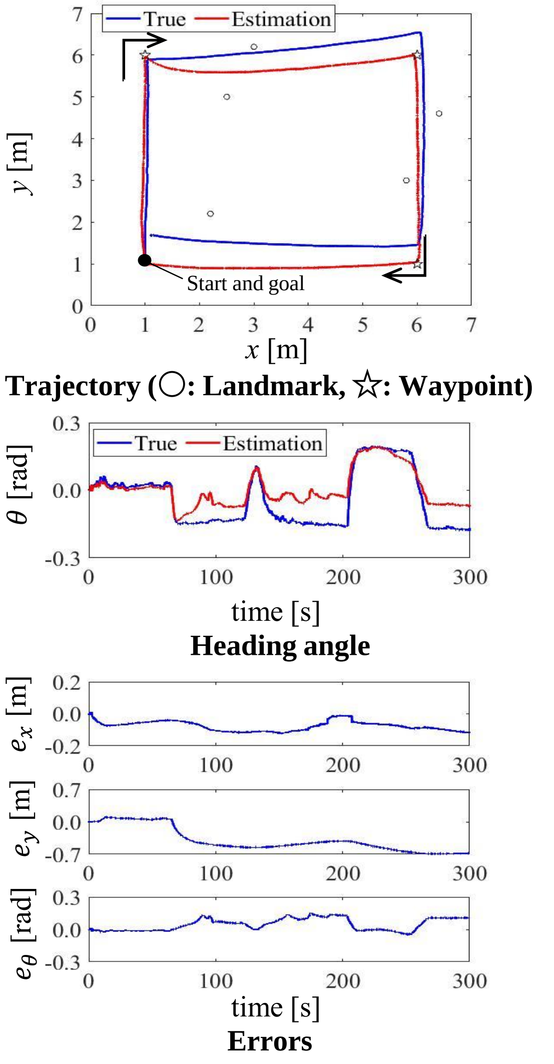
$$\alpha = 0.05$$

Performance indexes

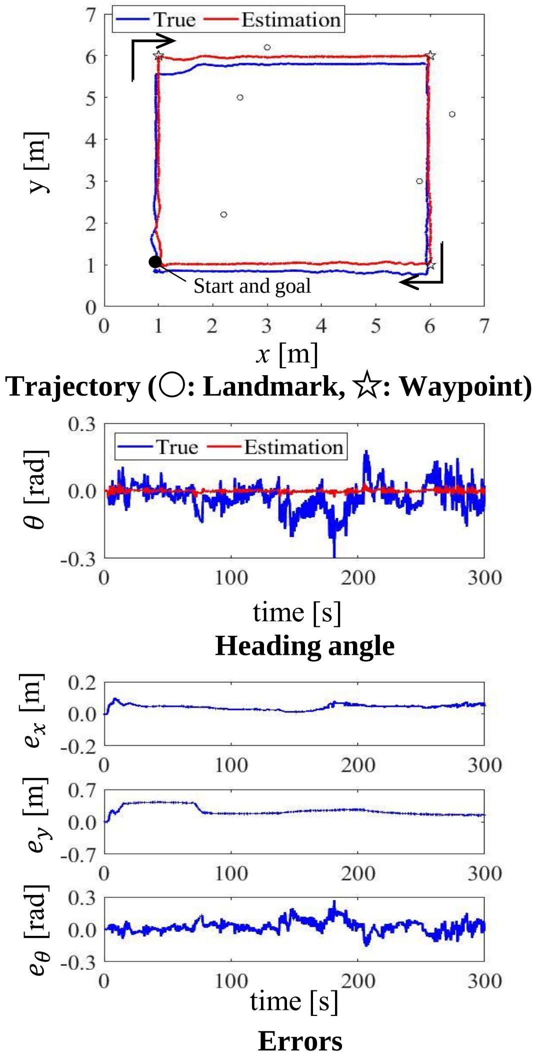
$$\begin{cases} IAE_{xy} = \sum_{t=0}^{k} (|e_x| + |e_y|) \\ IAE_\theta = \sum_{t=0}^{k} |e_\theta| \end{cases}$$

$$\begin{cases} MAE_{xy} = \max\{|e_x| + |e_y|\} \\ MAE_\theta = \max|e_\theta| \end{cases}$$

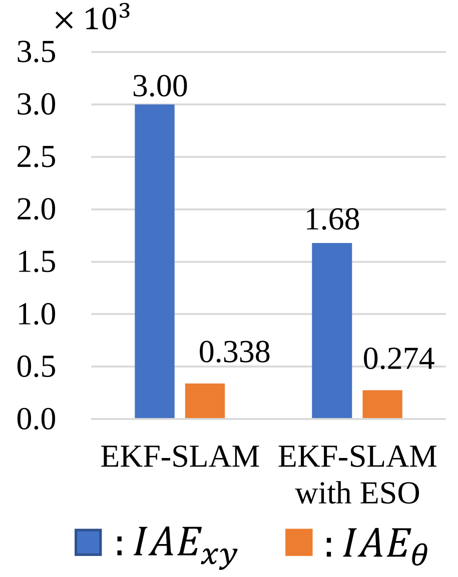
Results

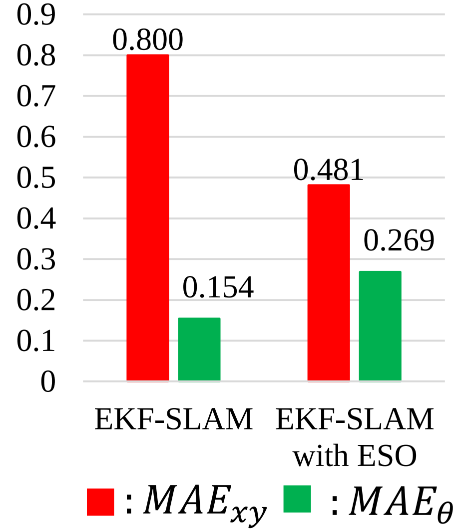


Estimation by **EKF-SLAM**

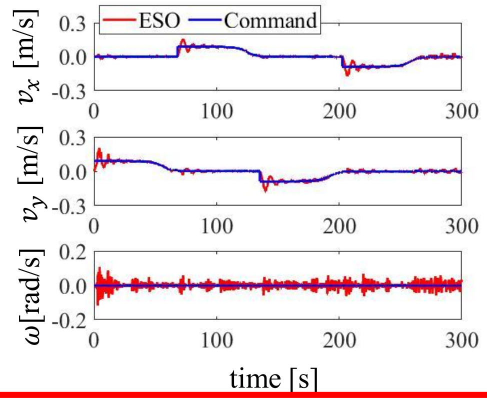


Estimation by **EKF-SLAM with ESO**

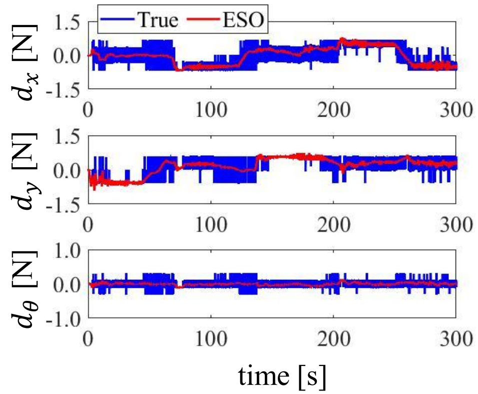




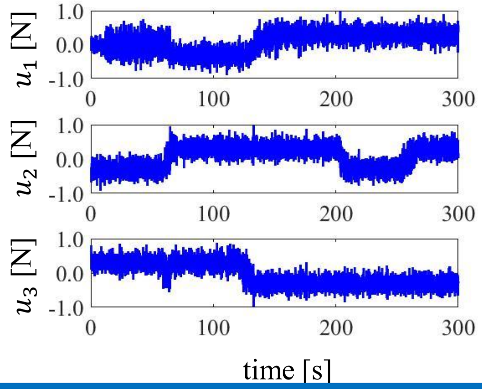
Results



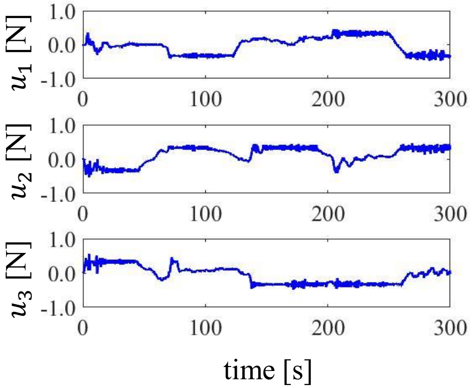
Estimation of the velocity by ESO



Estimation of the disturbances by ESO



Input of the OMR by **EKF-SLAM without ESO**



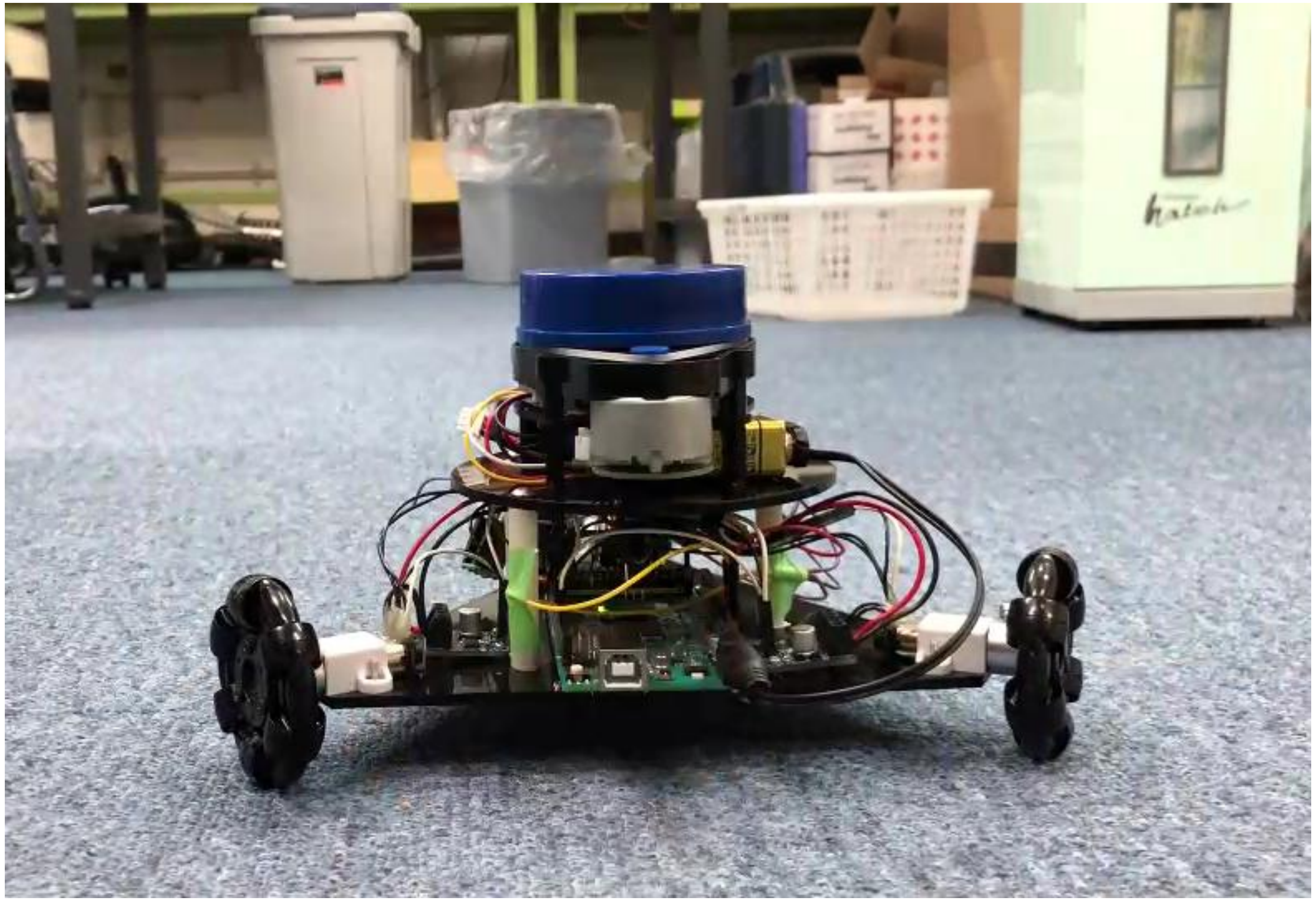
Input of the OMR by **EKF-SLAM with ESO**

Conclusion

- We proposed EKF-SLAM combined with ESO for OMR with three Omni-wheels.

- We verified the effectiveness of the proposed method by compare numerical simulation.

Future plan



Thank you for your attention.

Simulation condition -Gain-

EKF-SLAM

Control Gain K_P	8.0
Control Gain K_I	7.0
Control Gain K_D	-0.2

EKF-SLAM Combined with ESO

Control Gain K_P	0.1
Control Gain K_I	0.3
Control Gain K_D	-0.01
Observer Gain $\mathbf{l}_x$	$\mathrm{diag}([10 \quad 10 \quad 10])$
Observer Gain $\mathbf{l}_v$	$\mathrm{diag}([0.2 \quad 0.2 \quad 0.2])$
Observer Gain $\mathbf{l}_d$	$\mathrm{diag}([100 \quad 100 \quad 100])$

Design of ESO

$$\begin{cases} \tilde{\mathbf{x}} = \hat{\mathbf{x}} - \hat{\mathbf{h}}_x \\ \dot{\hat{\mathbf{h}}}_x = \hat{\mathbf{h}}_v + \mathbf{l}_x \tilde{\mathbf{x}} \\ \dot{\hat{\mathbf{h}}}_v = \mathbf{M}^{-1}\left(\mathbf{Bu} + \hat{\mathbf{h}}_d\right) + \mathbf{l}_v \tilde{\mathbf{x}} \\ \dot{\hat{\mathbf{h}}}_d = \mathbf{l}_d \tilde{\mathbf{x}} \end{cases}$$

PID Controller

$$\mathbf{u}_0 = \mathbf{K}_P\left(\mathbf{v}_c - \hat{\mathbf{h}}_v\right) + \mathbf{K}_I \sum_{t=0}^{k} \left(\mathbf{v}_c - \hat{\mathbf{h}}_v\right) - \mathbf{K}_D \dot{\hat{\mathbf{h}}}_v$$

Actual inputs

$$\mathbf{u} = \mathbf{B}^{-1}\left(\mathbf{u}_0 - \hat{\mathbf{h}}_d\right)$$

Simulation condition –Sensor condition-

Sensor condition

Distance error range [m]	0.05
Angle error range [deg]	0.1
Sensor range [m]	10

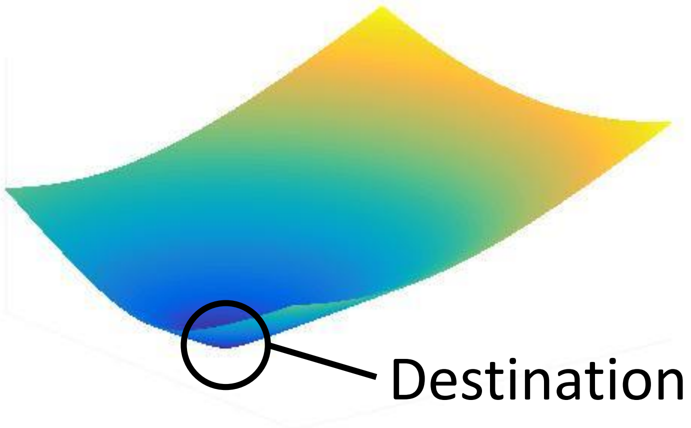
Measurement model

$$\mathbf{Z}(k) = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_j \end{bmatrix} = \begin{bmatrix} \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,1}\right) + \mathbf{r}(k) \\ \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,2}\right) + \mathbf{r}(k) \\ \vdots \\ \mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,j}\right) + \mathbf{r}(k) \end{bmatrix}$$

$$\mathrm{h}\left(\mathbf{x}(k), \mathbf{x}_{L,j}\right) = \begin{bmatrix} \sqrt{\left(x_{L,j} - x\right)^2 + \left(y_{L,j} - y\right)^2} \\ \arctan\left(\frac{y_{L,j} - y}{x_{L,j} - x}\right) - \theta \end{bmatrix}$$

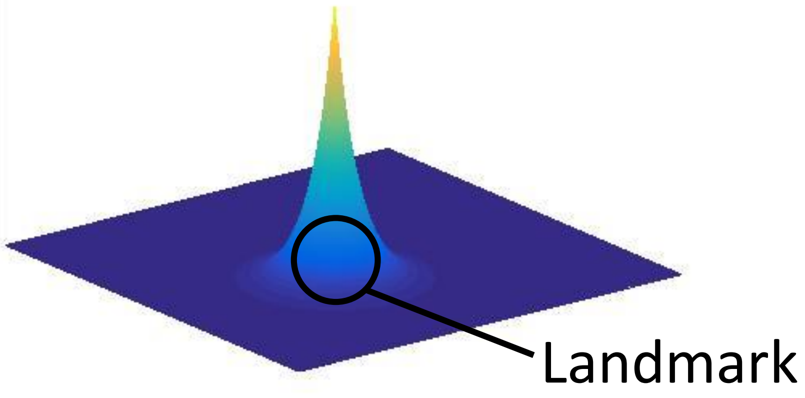
Potential function method

Steering potential function



$$U_S(\hat{\mathbf{x}}) = C_S\sqrt{|\hat{\mathbf{x}}_c|^2 + L_S}$$

Repulsing potential function



$$U_{R,j}(\mathbf{x}_j) = C_r \sum_j \exp\left(-\frac{|\hat{\mathbf{x}}_j|}{L_r}\right)$$

Velocity command

$$v_{xc} = -\frac{\partial U_S(\hat{\mathbf{x}})}{\partial x} - \frac{\partial U_{R,j}(\hat{\mathbf{x}}_j)}{\partial x} \qquad v_{yc} = -\frac{\partial U_S(\hat{\mathbf{x}})}{\partial y} - \frac{\partial U_{R,j}(\hat{\mathbf{x}}_j)}{\partial y}$$